Blur from Scanning System

Initialize Packages

```
Needs@"Statistics`DataManipulation`"D
Needs@"Graphics`Graphics`"D
Needs@"Statistics`MultiDescriptiveStatistics`"D
```

Beam Propagation: ABCD Matrices

These operators define the changes to a vector {x,x'} under Gaussian optics (paraxial case).

```
trans[d_] := {{1,d},{0,1}}
refr[n1_,n2_] := {{1,0},{0,n1/n2}}
lens[f_] := {{1,0},{-1/f,1}}
```

```
Print@MatrixForm@trans@dDD, ".", MatrixForm@8x, x1<D, " = ",
 MatrixForm@trans@dD . 8x, x1<DD
```

$$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} . \begin{pmatrix} x \\ x1 \end{pmatrix} = \begin{pmatrix} x + d\,x1 \\ x1 \end{pmatrix}$$

As expected. Hence, if we have a lens with focal length f and two propagation lengths (D-P) and P we have a global matrix:

```
prop = trans@pD . Hlens@fD . trans@d - pDL
```

$$\left\{\left\{1 - \frac{p}{f},\ d - p + \left(1 - \frac{d-p}{f}\right) p\right\},\ \left\{-\frac{1}{f},\ 1 - \frac{d-p}{f}\right\}\right\}$$

In matrix form:

```
MatrixForm@propD
```

$$\begin{pmatrix} 1 - \frac{p}{f} & d - p + \left(1 - \frac{d-p}{f}\right) p \\ -\frac{1}{f} & 1 - \frac{d-p}{f} \end{pmatrix}$$

Hence, for a starting point {x,x1} we obtain:

```
out = prop . 8x, x1<
```

$$9 \mathrm{I} 1 - \frac{p}{f} \mathrm{M}\, x + \mathrm{J} d - p + \mathrm{J} 1 - \frac{d - p}{f} \mathrm{N}\, p \mathrm{N}\, x1,\ -\frac{x}{f} + \mathrm{J} 1 - \frac{d - p}{f} \mathrm{N}\, x1 =$$

```
MatrixForm@Simplify@outDD
```

$$\begin{matrix} \frac{f\,\mathrm{H}x + d\,x1\mathrm{L} + p\,\mathrm{H} - x - d\,x1 + p\,x1\mathrm{L}}{f} \\ \frac{-x + \mathrm{H} - d + f + p\mathrm{L}\,x1}{f} \end{matrix}$$

The discussion becomes complicated when the distances p,d-p are not conjugate. To be at a focal position, the output should be independent of the angle variable x1.

Let mx, mx1 be the avergae values of the position and directions, sx and sx1 the standard deviations. Since the operators are linear, the averages mx, mx1 will transform like the rays:

```
Clear@mx, mx1D
mxout = prop . 8mx, mx1<
```

$$9 mx\ \mathrm{I} 1 - \frac{p}{f} \mathrm{M} + mx1\ \mathrm{J} d - p + \mathrm{J} 1 - \frac{d - p}{f} \mathrm{N}\, p \mathrm{N},\ -\frac{mx}{f} + mx1\ \mathrm{J} 1 - \frac{d - p}{f} \mathrm{N} =$$

```
MatrixForm@Simplify@mxoutDD
```

$$\begin{matrix} \frac{f\,\mathrm{H}mx + d\,mx1\mathrm{L} + p\,\mathrm{H} - mx - d\,mx1 + mx1\,p\mathrm{L}}{f} \\ \frac{-mx + mx1\,\mathrm{H} - d + f + p\mathrm{L}}{f} \end{matrix}$$

What about the standard deviation? We can deal with it by considering the special case of a Gaussian distribution. If we have a distribution:

■ Propagation of a Gaussian beam

```
Clear@g, x, y, mx, mx1, sx, sx1D
```

```
x = Hprop.8x0, y0<L@@1DD
y = Hprop.8x0, y0<L@@2DD
g = Hx - mxL^2 • 2 • sx^2 + Hy - myL^2 • 2 • sy^2
```

$$\text{I}1-\frac{p}{f}\text{M}\,x0+\text{J}d-p+\text{J}1-\frac{d-p}{f}\text{N}\,p\text{N}\,y0$$

$$-\frac{x0}{f}+\text{J}1-\frac{d-p}{f}\text{N}\,y0$$

$$\frac{\text{H}-my-\frac{x0}{f}+\text{H}1-\frac{d-p}{f}\text{L}\,y0\text{L}^2}{2\,sy^2}+\frac{\text{H}-mx+\text{H}1-\frac{p}{f}\text{L}\,x0+\text{H}d-p+\text{H}1-\frac{d-p}{f}\text{L}\,p\text{L}\,y0\text{L}^2}{2\,sx^2}$$

```
gstep1 = Expand@gD
```

$$\frac{mx^2}{2\,sx^2}+\frac{my^2}{2\,sy^2}-\frac{mx\,x0}{sx^2}+\frac{mx\,p\,x0}{f\,sx^2}+\frac{my\,x0}{f\,sy^2}+\frac{x0^2}{2\,sx^2}-\frac{p\,x0^2}{f\,sx^2}+$$
$$\frac{p^2\,x0^2}{2\,f^2\,sx^2}+\frac{x0^2}{2\,f^2\,sy^2}-\frac{d\,mx\,y0}{sx^2}+\frac{d\,mx\,p\,y0}{f\,sx^2}-\frac{mx\,p^2\,y0}{f\,sx^2}-\frac{my\,y0}{sy^2}+\frac{d\,my\,y0}{f\,sy^2}-$$
$$\frac{my\,p\,y0}{f\,sy^2}+\frac{d\,x0\,y0}{sx^2}-\frac{2\,d\,p\,x0\,y0}{f\,sx^2}+\frac{d\,p^2\,x0\,y0}{f^2\,sx^2}+\frac{p^2\,x0\,y0}{f\,sx^2}-\frac{p^3\,x0\,y0}{f^2\,sx^2}+$$
$$\frac{d\,x0\,y0}{f^2\,sy^2}-\frac{x0\,y0}{f\,sy^2}-\frac{p\,x0\,y0}{f^2\,sy^2}+\frac{d^2\,y0^2}{2\,sx^2}-\frac{d^2\,p\,y0^2}{f\,sx^2}+\frac{d^2\,p^2\,y0^2}{2\,f^2\,sx^2}+\frac{d\,p^2\,y0^2}{f\,sx^2}-$$
$$\frac{d\,p^3\,y0^2}{f^2\,sx^2}+\frac{p^4\,y0^2}{2\,f^2\,sx^2}+\frac{y0^2}{2\,sy^2}+\frac{d^2\,y0^2}{2\,f^2\,sy^2}-\frac{d\,y0^2}{f\,sy^2}-\frac{d\,p\,y0^2}{f^2\,sy^2}+\frac{p\,y0^2}{f\,sy^2}+\frac{p^2\,y0^2}{2\,f^2\,sy^2}$$

```
Clear@x, yD
```

```
ellipse = ReplaceAll@gstep1, 8x0 -> x, y0 -> y<D
```

$$\frac{mx^2}{2\,sx^2}+\frac{my^2}{2\,sy^2}-\frac{mx\,x}{sx^2}+\frac{mx\,p\,x}{f\,sx^2}+\frac{my\,x}{f\,sy^2}+\frac{x^2}{2\,sx^2}-\frac{p\,x^2}{f\,sx^2}+\frac{p^2\,x^2}{2\,f^2\,sx^2}+\frac{x^2}{2\,f^2\,sy^2}-\frac{d\,mx\,y}{sx^2}+\frac{d\,mx\,p\,y}{f\,sx^2}-\frac{mx\,p^2\,y}{f\,sx^2}-\frac{my\,y}{sy^2}+\frac{d\,my\,y}{f\,sy^2}-\frac{my\,p\,y}{f\,sy^2}+\frac{d\,x\,y}{sx^2}-\frac{2\,d\,p\,x\,y}{f\,sx^2}+\frac{d\,p^2\,x\,y}{f^2\,sx^2}+\frac{p^2\,x\,y}{f\,sx^2}-\frac{p^3\,x\,y}{f^2\,sx^2}+\frac{d\,x\,y}{f^2\,sy^2}-\frac{x\,y}{f\,sy^2}-\frac{p\,x\,y}{f^2\,sy^2}+\frac{d^2\,y^2}{2\,sx^2}-\frac{d^2\,p\,y^2}{f\,sx^2}+\frac{d^2\,p^2\,y^2}{2\,f^2\,sx^2}+\frac{d\,p^2\,y^2}{f\,sx^2}-\frac{d\,p^3\,y^2}{f^2\,sx^2}+\frac{p^4\,y^2}{2\,f^2\,sx^2}+\frac{y^2}{2\,sy^2}+\frac{d^2\,y^2}{2\,f^2\,sy^2}-\frac{d\,y^2}{f\,sy^2}-\frac{d\,p\,y^2}{f^2\,sy^2}+\frac{p\,y^2}{f\,sy^2}+\frac{p^2\,y^2}{2\,f^2\,sy^2}$$

```
TraditionalForm@Simplify@ellipseDD
```

$$\frac{1}{2\,f^2\,sx^2\,sy^2}\,((my^2\,sx^2 \quad y^2\,sx^2 \quad 2\,my\,y\,sx^2 \quad mx^2\,sy^2 \quad sy^2\,x^2 \quad d^2\,sy^2\,y^2 \quad 2\,mx\,sy^2\,x \quad 2\,d\,mx\,sy^2\,y \quad 2\,d\,sy^2\,x\,y)\,f^2 \quad 2\,(x \quad (d \quad p)\,y)\,(\quad my\,sx^2 \quad y\,sx^2 \quad mx\,p\,sy^2 \quad p\,sy^2\,x \quad d\,p\,sy^2\,y)\,f \quad (sx^2 \quad p^2\,sy^2)\,(x \quad (d \quad p)\,y)^2)$$

Note: it is important that the next cells be executed as a group, because of some recursion:

```
cx0 = Coefficient@ellipse, x, 0D
c00 = Coefficient@c0, y, 0D
cx = Coefficient@ellipse, x D
cxy = Coefficient@cx, yD
cx = Simplify@Expand@cx – cxy yDD
cy = Coefficient@ellipse, yD
cy = Simplify@Expand@cy – cxy xDD
cx2 = Coefficient@ellipse, x^2D
cy2 = Coefficient@ellipse, y^2D
```

$$\frac{mx^2}{2\,sx^2}+\frac{my^2}{2\,sy^2}-\frac{d\,mx\,y}{sx^2}+\frac{d\,mx\,p\,y}{f\,sx^2}-\frac{mx\,p^2\,y}{f\,sx^2}-\frac{my\,y}{sy^2}+\frac{d\,my\,y}{f\,sy^2}-\frac{my\,p\,y}{f\,sy^2}+\frac{d^2\,y^2}{2\,sx^2}-\frac{d^2\,p\,y^2}{f\,sx^2}+\frac{d^2\,p^2\,y^2}{2\,f^2\,sx^2}+\frac{d\,p^2\,y^2}{f\,sx^2}-\frac{d\,p^3\,y^2}{f^2\,sx^2}+\frac{p^4\,y^2}{2\,f^2\,sx^2}+\frac{y^2}{2\,sy^2}+\frac{d^2\,y^2}{2\,f^2\,sy^2}-\frac{d\,y^2}{f\,sy^2}-\frac{d\,p\,y^2}{f^2\,sy^2}+\frac{p\,y^2}{f\,sy^2}+\frac{p^2\,y^2}{2\,f^2\,sy^2}$$

$$\frac{mx^2}{2\,sx^2}+\frac{my^2}{2\,sy^2}$$

$$-\frac{mx}{sx^2}+\frac{mx\,p}{f\,sx^2}+\frac{my}{f\,sy^2}+\frac{d\,y}{sx^2}-\frac{2\,d\,p\,y}{f\,sx^2}+\frac{d\,p^2\,y}{f^2\,sx^2}+\frac{p^2\,y}{f\,sx^2}-\frac{p^3\,y}{f^2\,sx^2}+\frac{d\,y}{f^2\,sy^2}-\frac{y}{f\,sy^2}-\frac{p\,y}{f^2\,sy^2}$$

$$\frac{d}{sx^2}-\frac{2\,d\,p}{f\,sx^2}+\frac{d\,p^2}{f^2\,sx^2}+\frac{p^2}{f\,sx^2}-\frac{p^3}{f^2\,sx^2}+\frac{d}{f^2\,sy^2}-\frac{1}{f\,sy^2}-\frac{p}{f^2\,sy^2}$$

$$\frac{\frac{mx\,H-f+pL}{sx^2}+\frac{my}{sy^2}}{f}$$

$$-\frac{d\,mx}{sx^2}+\frac{d\,mx\,p}{f\,sx^2}-\frac{mx\,p^2}{f\,sx^2}-\frac{my}{sy^2}+\frac{d\,my}{f\,sy^2}-\frac{my\,p}{f\,sy^2}+\frac{d\,x}{sx^2}-\frac{2\,d\,p\,x}{f\,sx^2}+\frac{d\,p^2\,x}{f^2\,sx^2}+\frac{p^2\,x}{f\,sx^2}-\frac{p^3\,x}{f^2\,sx^2}+\frac{d\,x}{f^2\,sy^2}-\frac{x}{f\,sy^2}-\frac{p\,x}{f^2\,sy^2}$$

$$-\frac{f\,my\,sx^2+p\,Hmy\,sx^2+mx\,p\,sy^2L-d\,Hmy\,sx^2+mx\,H-f+pL\,sy^2L}{f\,sx^2\,sy^2}$$

$$\frac{1}{2\,sx^2}-\frac{p}{f\,sx^2}+\frac{p^2}{2\,f^2\,sx^2}+\frac{1}{2\,f^2\,sy^2}$$

$$\frac{d^2}{2\,sx^2}-\frac{d^2\,p}{f\,sx^2}+\frac{d^2\,p^2}{2\,f^2\,sx^2}+\frac{d\,p^2}{f\,sx^2}-\frac{d\,p^3}{f^2\,sx^2}+\frac{p^4}{2\,f^2\,sx^2}+\frac{1}{2\,sy^2}+\frac{d^2}{2\,f^2\,sy^2}-\frac{d}{f\,sy^2}-\frac{d\,p}{f^2\,sy^2}+\frac{p}{f\,sy^2}+\frac{p^2}{2\,f^2\,sy^2}$$

The next equation must yield zero if the coefficients are correct:

```
Simplify@ellipse - c00 - cx x - cy y - cxy x y - cx2 x^2 - cy2 y^2D
```

```
0
```

■ Correlated distribution

Let us now compare with the ellipse of a gaussian of correlation coefficient r

```
Clear@u, v, mu, mv, sv, suD
corr = Expand@1 • 2 • H1 – r^2L HHu – muL^2 • su^2
 + Hv – mvL^2 • sv^2
 – 2 r Hu – muL Hv – mvL • su • svLD
```

$$\frac{mu^2}{2\,(1-r^2)\,su^2}+\frac{mv^2}{2\,(1-r^2)\,sv^2}-\frac{mu\,mv\,r}{(1-r^2)\,su\,sv}-\frac{mu\,u}{(1-r^2)\,su^2}+\frac{mv\,r\,u}{(1-r^2)\,su\,sv}+\frac{u^2}{2\,(1-r^2)\,su^2}-\frac{mv\,v}{(1-r^2)\,sv^2}+\frac{mu\,r\,v}{(1-r^2)\,su\,sv}-\frac{r\,u\,v}{(1-r^2)\,su\,sv}+\frac{v^2}{2\,(1-r^2)\,sv^2}$$

```
bu0 = Coefficient@corr, u, 0D
b00 = Coefficient@bx0, v, 0D
bu = Coefficient@corr, u, 1 D
buv = Coefficient@bu, vD
bu = Simplify@Expand@bu – buv DD
bv = Coefficient@corr, yD
bv = Simplify@Expand@by – bxy xDD
bu2 = Coefficient@corr, x^2D
bv2 = Coefficient@corr, y^2D
```

$$\frac{mx1^2}{2\,H1 - r^2 L\, sx1^2} + \frac{my1^2}{2\,H1 - r^2 L\, sy1^2} - \frac{mx1\, my1\, r}{H1 - r^2 L\, sx1\, sy1} - \frac{my1\, y}{H1 - r^2 L\, sy1^2} + \frac{mx1\, r\, y}{H1 - r^2 L\, sx1\, sy1} + \frac{y^2}{2\,H1 - r^2 L\, sy1^2}$$

$$\frac{mx1^2}{2\,H1 - r^2 L\, sx1^2} + \frac{my1^2}{2\,H1 - r^2 L\, sy1^2} - \frac{mx1\, my1\, r}{H1 - r^2 L\, sx1\, sy1}$$

$$-\frac{mx1}{H1 - r^2 L\, sx1^2} + \frac{my1\, r}{H1 - r^2 L\, sx1\, sy1} - \frac{r\, y}{H1 - r^2 L\, sx1\, sy1}$$

$$-\frac{r}{H1 - r^2 L\, sx1\, sy1}$$

$$\frac{-my1\, r\, sx1 + mx1\, sy1}{H-1 + r^2 L\, sx1^2\, sy1}$$

$$-\frac{my1}{H1 - r^2 L\, sy1^2} + \frac{mx1\, r}{H1 - r^2 L\, sx1\, sy1} - \frac{r\, x}{H1 - r^2 L\, sx1\, sy1}$$

$$\frac{my1\, sx1 - mx1\, r\, sy1}{H-1 + r^2 L\, sx1\, sy1^2}$$

$$\frac{1}{2\,H1 - r^2 L\, sx1^2}$$

$$\frac{1}{2\,H1 - r^2 L\, sy1^2}$$

Hence, we have the new coefficients:

```
sxnew2 =
synew2 =
```

If {x,x1} represent a pencil of rays, the distribution will have average values:

Scanning Operation

We start with a beam focused at the mask:

```
g@x_, y_D = 1 • 2 • H1 – r^2L
  HHx – mxL^2 • sx^2 –  2 r Hx – mxL H y – myL • sx • sy + Hy – myL^2 • sy^2L
```

$$\frac{\frac{(-mx+x)^2}{sx^2} - \frac{2\,r\,(-mx+x)\,(-my+y)}{sx\,sy} + \frac{(-my+y)^2}{sy^2}}{2\,(1-r^2)}$$

Note: we use y instead of x' for simplicity of notation.

Then we introduce the scanning operation:

```
gt@x_, y_, t_D = Expand@ReplaceAll@g@x, yD, 8x -> x + p t, y -> y + t<DD
at1 = Coefficient@gt@x, y, tD, tD
at2 = Coefficient@gt@x, y, tD, t^2D
at0 = Coefficient@gt@x, y, tD, t, 0D
```

$$\frac{mx^2}{2\,H1 - r^2L\,sx^2} + \frac{my^2}{2\,H1 - r^2L\,sy^2} - \frac{mx\,my\,r}{H1 - r^2L\,sx\,sy} - \frac{mx\,p\,t}{H1 - r^2L\,sx^2} - \frac{my\,t}{H1 - r^2L\,sy^2} + \frac{mx\,r\,t}{H1 - r^2L\,sx\,sy} + \frac{my\,p\,r\,t}{H1 - r^2L\,sx\,sy} + \frac{p^2\,t^2}{2\,H1 - r^2L\,sx^2} + \frac{t^2}{2\,H1 - r^2L\,sy^2} - \frac{p\,r\,t^2}{H1 - r^2L\,sx\,sy} - \frac{mx\,x}{H1 - r^2L\,sx^2} + \frac{my\,r\,x}{H1 - r^2L\,sx\,sy} + \frac{p\,t\,x}{H1 - r^2L\,sx^2} - \frac{r\,t\,x}{H1 - r^2L\,sx\,sy} + \frac{x^2}{2\,H1 - r^2L\,sx^2} - \frac{my\,y}{H1 - r^2L\,sy^2} + \frac{mx\,r\,y}{H1 - r^2L\,sx\,sy} + \frac{t\,y}{H1 - r^2L\,sy^2} - \frac{p\,r\,t\,y}{H1 - r^2L\,sx\,sy} - \frac{r\,x\,y}{H1 - r^2L\,sx\,sy} + \frac{y^2}{2\,H1 - r^2L\,sy^2}$$

$$-\frac{mx\,p}{H1 - r^2L\,sx^2} - \frac{my}{H1 - r^2L\,sy^2} + \frac{mx\,r}{H1 - r^2L\,sx\,sy} + \frac{my\,p\,r}{H1 - r^2L\,sx\,sy} + \frac{p\,x}{H1 - r^2L\,sx^2} - \frac{r\,x}{H1 - r^2L\,sx\,sy} + \frac{y}{H1 - r^2L\,sy^2} - \frac{p\,r\,y}{H1 - r^2L\,sx\,sy}$$

$$\frac{p^2}{2\,H1 - r^2L\,sx^2} + \frac{1}{2\,H1 - r^2L\,sy^2} - \frac{p\,r}{H1 - r^2L\,sx\,sy}$$

$$\frac{mx^2}{2\,H1 - r^2L\,sx^2} + \frac{my^2}{2\,H1 - r^2L\,sy^2} - \frac{mx\,my\,r}{H1 - r^2L\,sx\,sy} - \frac{mx\,x}{H1 - r^2L\,sx^2} + \frac{my\,r\,x}{H1 - r^2L\,sx\,sy} + \frac{x^2}{2\,H1 - r^2L\,sx^2} - \frac{my\,y}{H1 - r^2L\,sy^2} + \frac{mx\,r\,y}{H1 - r^2L\,sx\,sy} - \frac{r\,x\,y}{H1 - r^2L\,sx\,sy} + \frac{y^2}{2\,H1 - r^2L\,sy^2}$$

```
ggt@x_, y_, t_D = at0 + at1 t + at2 t^2
```

$$\frac{mx^2}{2\,H1 - r^2L\,sx^2} + \frac{my^2}{2\,H1 - r^2L\,sy^2} - \frac{mx\,my\,r}{H1 - r^2L\,sx\,sy} + J\frac{p^2}{2\,H1 - r^2L\,sx^2} + \frac{1}{2\,H1 - r^2L\,sy^2} - \frac{p\,r}{H1 - r^2L\,sx\,sy}N\,t^2 - \frac{mx\,x}{H1 - r^2L\,sx^2} + \frac{my\,r\,x}{H1 - r^2L\,sx\,sy} + \frac{x^2}{2\,H1 - r^2L\,sx^2} - \frac{my\,y}{H1 - r^2L\,sy^2} + \frac{mx\,r\,y}{H1 - r^2L\,sx\,sy} - \frac{r\,x\,y}{H1 - r^2L\,sx\,sy} + \frac{y^2}{2\,H1 - r^2L\,sy^2} + t\,J - \frac{mx\,p}{H1 - r^2L\,sx^2} - \frac{my}{H1 - r^2L\,sy^2} + \frac{mx\,r}{H1 - r^2L\,sx\,sy} + \frac{my\,p\,r}{H1 - r^2L\,sx\,sy} + \frac{p\,x}{H1 - r^2L\,sx^2} - \frac{r\,x}{H1 - r^2L\,sx\,sy} + \frac{y}{H1 - r^2L\,sy^2} - \frac{p\,r\,y}{H1 - r^2L\,sx\,sy}N$$

```
step1 = Integrate@Exp@-Ha0 + a1 t + a2 t^2LD, 8t, -Infinity, Infinity<D
```

$$IfARe@a1D > 0 \;\&\&\; Re@a2D > 0,\; \frac{E^{-a0+\frac{a1^2}{4\,a2}}\sqrt{\pi}}{\sqrt{a2}},\; à_{-\infty}^{\infty} E^{-a0-a1\,t-a2\,t^2}\,dt\,E$$

And we integrate over the scanning angle:

```
gg@x_, y_D = Integrate@Exp@- ggt@x, y, tDD, 8t, -Infinity, Infinity<D
```

$$IfAReA\frac{p^2}{2\,H1 - r^2L\,sx^2} + \frac{1}{2\,H1 - r^2L\,sy^2} - \frac{p\,r}{H1 - r^2L\,sx\,sy}E > 0 \;\&\&$$

$$ReA - \frac{mx\,p}{H1 - r^2L\,sx^2} - \frac{my}{H1 - r^2L\,sy^2} + \frac{mx\,r}{H1 - r^2L\,sx\,sy} + \frac{my\,p\,r}{H1 - r^2L\,sx\,sy} + \frac{p\,x}{H1 - r^2L\,sx^2} - \frac{r\,x}{H1 - r^2L\,sx\,sy} + \frac{y}{H1 - r^2L\,sy^2} - \frac{p\,r\,y}{H1 - r^2L\,sx\,sy}E > 0 \;\&\&$$

$$ReA\frac{mx\,p}{H1 - r^2L\,sx^2} + \frac{my}{H1 - r^2L\,sy^2} - \frac{mx\,r}{H1 - r^2L\,sx\,sy} - \frac{my\,p\,r}{H1 - r^2L\,sx\,sy} - \frac{p\,x}{H1 - r^2L\,sx^2} + \frac{r\,x}{H1 - r^2L\,sx\,sy} - \frac{y}{H1 - r^2L\,sy^2} + \frac{p\,r\,y}{H1 - r^2L\,sx\,sy}E > 0 \;\&\&$$

$$ReA - \frac{p^2}{2\,H1 - r^2L\,sx^2} - \frac{1}{2\,H1 - r^2L\,sy^2} + \frac{p\,r}{H1 - r^2L\,sx\,sy}E < 0 \;\&\&$$

$$ReA\frac{p^2}{2\,H-1 + r^2L\,sx^2} + \frac{1}{2\,H-1 + r^2L\,sy^2} - \frac{p\,r}{H-1 + r^2L\,sx\,sy}E < 0,$$

$$\left\{ E \wedge J - \frac{mx^2}{2\,H1 - r^2L\,sx^2} - \frac{my^2}{2\,H1 - r^2L\,sy^2} + \frac{mx\,my\,r}{H1 - r^2L\,sx\,sy} + \frac{mx\,x}{H1 - r^2L\,sx^2} - \frac{my\,r\,x}{H1 - r^2L\,sx\,sy} - \frac{x^2}{2\,H1 - r^2L\,sx^2} + \frac{my\,y}{H1 - r^2L\,sy^2} - \frac{mx\,r\,y}{H1 - r^2L\,sx\,sy} + \frac{r\,x\,y}{H1 - r^2L\,sx\,sy} - \frac{y^2}{2\,H1 - r^2L\,sy^2}N \right\}$$

$$\text{JE}^\wedge\text{JJ}-\frac{mx\,p}{H1-r^2L\,sx^2}-\frac{my}{H1-r^2L\,sy^2}+$$

$$\frac{mx\,r}{H1-r^2L\,sx\,sy}+\frac{my\,p\,r}{H1-r^2L\,sx\,sy}+\frac{p\,x}{H1-r^2L\,sx^2}-$$

$$\frac{r\,x}{H1-r^2L\,sx\,sy}+\frac{y}{H1-r^2L\,sy^2}-\frac{p\,r\,y}{H1-r^2L\,sx\,sy}\text{N}^\wedge 2\text{ ’}$$

$$\text{J4 J}\frac{p^2}{2\,H1-r^2L\,sx^2}+\frac{1}{2\,H1-r^2L\,sy^2}-\frac{p\,r}{H1-r^2L\,sx\,sy}\text{NNN}\,\pi$$

$$\text{J}-\frac{mx\,p}{H1-r^2L\,sx^2}-\frac{my}{H1-r^2L\,sy^2}+\frac{mx\,r}{H1-r^2L\,sx\,sy}+\frac{my\,p\,r}{H1-r^2L\,sx\,sy}+$$

$$\frac{p\,x}{H1-r^2L\,sx^2}-\frac{r\,x}{H1-r^2L\,sx\,sy}+\frac{y}{H1-r^2L\,sy^2}-\frac{p\,r\,y}{H1-r^2L\,sx\,sy}\text{NN “}$$

$$2\,\$\sqrt{\frac{p^2}{2\,H1-r^2L\,sx^2}+\frac{1}{2\,H1-r^2L\,sy^2}-\frac{p\,r}{H1-r^2L\,sx\,sy}}\;-$$

$$\text{E}^\wedge\text{JJ}-\frac{mx\,p}{H1-r^2L\,sx^2}-\frac{my}{H1-r^2L\,sy^2}+$$

$$\frac{mx\,r}{H1-r^2L\,sx\,sy}+\frac{my\,p\,r}{H1-r^2L\,sx\,sy}+\frac{p\,x}{H1-r^2L\,sx^2}-$$

$$\frac{r\,x}{H1-r^2L\,sx\,sy}+\frac{y}{H1-r^2L\,sy^2}-\frac{p\,r\,y}{H1-r^2L\,sx\,sy}\text{N}^\wedge 2\text{ ’}$$

$$\text{J4 J}\frac{p^2}{2\,H1-r^2L\,sx^2}+\frac{1}{2\,H1-r^2L\,sy^2}-\frac{p\,r}{H1-r^2L\,sx\,sy}\text{NNN}\,\pi$$

$$\text{J}-\frac{mx\,p}{H1-r^2L\,sx^2}-\frac{my}{H1-r^2L\,sy^2}+\frac{mx\,r}{H1-r^2L\,sx\,sy}+\frac{my\,p\,r}{H1-r^2L\,sx\,sy}+$$

$$\frac{p\,x}{H1-r^2L\,sx^2}-\frac{r\,x}{H1-r^2L\,sx\,sy}+\frac{y}{H1-r^2L\,sy^2}-\frac{p\,r\,y}{H1-r^2L\,sx\,sy}\text{N}$$

$$\text{ErfA}\frac{my\,sx^2\;\text{"}\sqrt{\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}}{\bullet!!!\,2\;H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}-$$

$$\frac{my\,r^2\,sx^2\;\text{"}\sqrt{\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}}{\bullet!!!\,2\;H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}-$$

$$\frac{mx\,r\,sx\,sy\;\text{"}\sqrt{\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}}{\bullet!!!\,2\;H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}-$$

$$\frac{my\,p\,r\,sx\,sy\;\text{"}\sqrt{\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}}{\bullet!!!\,2\;H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}+$$

$$\frac{mx\,r^3\,sx\,sy\;\text{"}\sqrt{\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}}{\bullet!!!\,2\;H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}+$$

$$\frac{my\,p\,r^3\,sx\,sy\;\text{"}\sqrt{\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}}{\bullet!!!\,2\;H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}+$$

$$\frac{mx\,p\,sy^2\;\text{"}\sqrt{\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}}{\bullet!!!\,2\;H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}-$$

$$\frac{mx\ p\ r^2\ sy^2\ "\frac{sx^2\ 2\ p\ r\ sx\ sy\ p^2\ sy^2}{H-1+r^2L\ sx^2\ sy^2}}{•!!!2\ H1-r^2L\ H-sx^2+2\ p\ r\ sx\ sy-p^2\ sy^2L}+$$

$$\frac{r\ sx\ sy\ "\frac{sx^2\ 2\ p\ r\ sx\ sy\ p^2\ sy^2}{H-1+r^2L\ sx^2\ sy^2}\ x}{•!!!2\ H1-r^2L\ H-sx^2+2\ p\ r\ sx\ sy-p^2\ sy^2L}-$$

$$\frac{r^3\ sx\ sy\ "\frac{sx^2\ 2\ p\ r\ sx\ sy\ p^2\ sy^2}{H-1+r^2L\ sx^2\ sy^2}\ x}{•!!!2\ H1-r^2L\ H-sx^2+2\ p\ r\ sx\ sy-p^2\ sy^2L}-$$

$$\frac{p\ sy^2\ "\frac{sx^2\ 2\ p\ r\ sx\ sy\ p^2\ sy^2}{H-1+r^2L\ sx^2\ sy^2}\ x}{•!!!2\ H1-r^2L\ H-sx^2+2\ p\ r\ sx\ sy-p^2\ sy^2L}+$$

$$\frac{p\ r^2\ sy^2\ "\frac{sx^2\ 2\ p\ r\ sx\ sy\ p^2\ sy^2}{H-1+r^2L\ sx^2\ sy^2}\ x}{•!!!2\ H1-r^2L\ H-sx^2+2\ p\ r\ sx\ sy-p^2\ sy^2L}-$$

$$\frac{sx^2\ "\frac{sx^2\ 2\ p\ r\ sx\ sy\ p^2\ sy^2}{H-1+r^2L\ sx^2\ sy^2}\ y}{•!!!2\ H1-r^2L\ H-sx^2+2\ p\ r\ sx\ sy-p^2\ sy^2L}+$$

$$\frac{r^2\ sx^2\ "\frac{sx^2\ 2\ p\ r\ sx\ sy\ p^2\ sy^2}{H-1+r^2L\ sx^2\ sy^2}\ y}{•!!!2\ H1-r^2L\ H-sx^2+2\ p\ r\ sx\ sy-p^2\ sy^2L}+$$

$$\frac{p\ r\ sx\ sy\ "\frac{sx^2\ 2\ p\ r\ sx\ sy\ p^2\ sy^2}{H-1+r^2L\ sx^2\ sy^2}\ y}{•!!!2\ H1-r^2L\ H-sx^2+2\ p\ r\ sx\ sy-p^2\ sy^2L}-$$

$$\frac{p\ r^3\ sx\ sy\ "\frac{sx^2\ 2\ p\ r\ sx\ sy\ p^2\ sy^2}{H-1+r^2L\ sx^2\ sy^2}\ y}{•!!!2\ H1-r^2L\ H-sx^2+2\ p\ r\ sx\ sy-p^2\ sy^2L}E\ “$$

$$2\ \$\frac{p^2}{2\ H1-r^2L\ sx^2}+\frac{1}{2\ H1-r^2L\ sy^2}-\frac{p\ r}{H1-r^2L\ sx\ sy}\ “$$

$$J\ •!!!\pi\ J-\frac{mx\ p}{H1-r^2L\ sx^2}-\frac{my}{H1-r^2L\ sy^2}+\frac{mx\ r}{H1-r^2L\ sx\ sy}+\frac{my\ p\ r}{H1-r^2L\ sx\ sy}+$$
$$\frac{p\ x}{H1-r^2L\ sx^2}-\frac{r\ x}{H1-r^2L\ sx\ sy}+\frac{y}{H1-r^2L\ sy^2}-\frac{p\ r\ y}{H1-r^2L\ sx\ sy}NN+$$

$$E\ \hat{}\ J-\frac{mx^2}{2\ H1-r^2L\ sx^2}-\frac{my^2}{2\ H1-r^2L\ sy^2}+$$
$$\frac{mx\ my\ r}{H1-r^2L\ sx\ sy}+\frac{mx\ x}{H1-r^2L\ sx^2}-\frac{my\ r\ x}{H1-r^2L\ sx\ sy}-\frac{x^2}{2\ H1-r^2L\ sx^2}+$$
$$\frac{my\ y}{H1-r^2L\ sy^2}-\frac{mx\ r\ y}{H1-r^2L\ sx\ sy}+\frac{r\ x\ y}{H1-r^2L\ sx\ sy}-\frac{y^2}{2\ H1-r^2L\ sy^2}N$$

$$JE\ \hat{}\ JJ\frac{mx\ p}{H1-r^2L\ sx^2}+\frac{my}{H1-r^2L\ sy^2}-$$
$$\frac{mx\ r}{H1-r^2L\ sx\ sy}-\frac{my\ p\ r}{H1-r^2L\ sx\ sy}-\frac{p\ x}{H1-r^2L\ sx^2}+$$
$$\frac{r\ x}{H1-r^2L\ sx\ sy}-\frac{y}{H1-r^2L\ sy^2}+\frac{p\ r\ y}{H1-r^2L\ sx\ sy}N\ \hat{}\ 2\ ’$$

$$J4\ J\frac{p^2}{2\ H1-r^2L\ sx^2}+\frac{1}{2\ H1-r^2L\ sy^2}-\frac{p\ r}{H1-r^2L\ sx\ sy}NNN\ \pi$$

$$J\frac{mx\,p}{H1-r^2L\,sx^2}+\frac{my}{H1-r^2L\,sy^2}-\frac{mx\,r}{H1-r^2L\,sx\,sy}-\frac{my\,p\,r}{H1-r^2L\,sx\,sy}-$$

$$\frac{p\,x}{H1-r^2L\,sx^2}+\frac{r\,x}{H1-r^2L\,sx\,sy}-\frac{y}{H1-r^2L\,sy^2}+\frac{p\,r\,y}{H1-r^2L\,sx\,sy}NN\text{“}$$

$$2\,\$\frac{p^2}{2\,H1-r^2L\,sx^2}+\frac{1}{2\,H1-r^2L\,sy^2}-\frac{p\,r}{H1-r^2L\,sx\,sy}+$$

$$E\,^\wedge JJ\frac{mx\,p}{H1-r^2L\,sx^2}+\frac{my}{H1-r^2L\,sy^2}-$$

$$\frac{mx\,r}{H1-r^2L\,sx\,sy}-\frac{my\,p\,r}{H1-r^2L\,sx\,sy}-\frac{p\,x}{H1-r^2L\,sx^2}+$$

$$\frac{r\,x}{H1-r^2L\,sx\,sy}-\frac{y}{H1-r^2L\,sy^2}+\frac{p\,r\,y}{H1-r^2L\,sx\,sy}N\,^\wedge 2\,\text{’}$$

$$J4\,J\frac{p^2}{2\,H1-r^2L\,sx^2}+\frac{1}{2\,H1-r^2L\,sy^2}-\frac{p\,r}{H1-r^2L\,sx\,sy}NNN\,\pi$$

$$J\frac{mx\,p}{H1-r^2L\,sx^2}+\frac{my}{H1-r^2L\,sy^2}-\frac{mx\,r}{H1-r^2L\,sx\,sy}-\frac{my\,p\,r}{H1-r^2L\,sx\,sy}-$$

$$\frac{p\,x}{H1-r^2L\,sx^2}+\frac{r\,x}{H1-r^2L\,sx\,sy}-\frac{y}{H1-r^2L\,sy^2}+\frac{p\,r\,y}{H1-r^2L\,sx\,sy}N$$

$$ErfA\frac{my\,sx^2\;"\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}{\bullet 2\,H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}-$$

$$\frac{my\,r^2\,sx^2\;"\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}{\bullet 2\,H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}-$$

$$\frac{mx\,r\,sx\,sy\;"\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}{\bullet 2\,H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}-$$

$$\frac{my\,p\,r\,sx\,sy\;"\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}{\bullet 2\,H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}+$$

$$\frac{mx\,r^3\,sx\,sy\;"\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}{\bullet 2\,H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}+$$

$$\frac{my\,p\,r^3\,sx\,sy\;"\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}{\bullet 2\,H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}+$$

$$\frac{mx\,p\,sy^2\;"\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}{\bullet 2\,H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}-$$

$$\frac{mx\,p\,r^2\,sy^2\;"\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}{\bullet 2\,H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}+$$

$$\frac{r\,sx\,sy\;"\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}\;x}{\bullet 2\,H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}-$$

$$\frac{r^3\,sx\,sy\;"\frac{sx^2-2\,p\,r\,sx\,sy+p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}\;x}{\bullet 2\,H1-r^2L\,H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}-$$

$$\frac{p\,sy^2\ \text{"}\!\sqrt{\frac{sx^2\ 2\,p\,r\,sx\,sy\ p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}\ x}{\bullet!!!\ 2\ H1-r^2L\ H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}+$$

$$\frac{p\,r^2\,sy^2\ \text{"}\!\sqrt{\frac{sx^2\ 2\,p\,r\,sx\,sy\ p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}\ x}{\bullet!!!\ 2\ H1-r^2L\ H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}-$$

$$\frac{sx^2\ \text{"}\!\sqrt{\frac{sx^2\ 2\,p\,r\,sx\,sy\ p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}\ y}{\bullet!!!\ 2\ H1-r^2L\ H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}+$$

$$\frac{r^2\,sx^2\ \text{"}\!\sqrt{\frac{sx^2\ 2\,p\,r\,sx\,sy\ p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}\ y}{\bullet!!!\ 2\ H1-r^2L\ H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}+$$

$$\frac{p\,r\,sx\,sy\ \text{"}\!\sqrt{\frac{sx^2\ 2\,p\,r\,sx\,sy\ p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}\ y}{\bullet!!!\ 2\ H1-r^2L\ H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}-$$

$$\frac{p\,r^3\,sx\,sy\ \text{"}\!\sqrt{\frac{sx^2\ 2\,p\,r\,sx\,sy\ p^2\,sy^2}{H-1+r^2L\,sx^2\,sy^2}}\ y}{\bullet!!!\ 2\ H1-r^2L\ H-sx^2+2\,p\,r\,sx\,sy-p^2\,sy^2L}E \text{“}$$

$$2\ \$\sqrt{\frac{p^2}{2\,H1-r^2L\,sx^2}+\frac{1}{2\,H1-r^2L\,sy^2}-\frac{p\,r}{H1-r^2L\,sx\,sy}}\ \text{“}$$

$$J\ \bullet!!!\ \pi\ J\frac{mx\,p}{H1-r^2L\,sx^2}+\frac{my}{H1-r^2L\,sy^2}-\frac{mx\,r}{H1-r^2L\,sx\,sy}-\frac{my\,p\,r}{H1-r^2L\,sx\,sy}-$$
$$\frac{p\,x}{H1-r^2L\,sx^2}+\frac{r\,x}{H1-r^2L\,sx\,sy}-\frac{y}{H1-r^2L\,sy^2}+\frac{p\,r\,y}{H1-r^2L\,sx\,sy}NN,$$

$$\grave{a}_{-\infty}^{\infty}E\text{^}J-\frac{mx^2}{2\,H1-r^2L\,sx^2}-\frac{my^2}{2\,H1-r^2L\,sy^2}+\frac{mx\,my\,r}{H1-r^2L\,sx\,sy}-$$
$$J\frac{p^2}{2\,H1-r^2L\,sx^2}+\frac{1}{2\,H1-r^2L\,sy^2}-\frac{p\,r}{H1-r^2L\,sx\,sy}N\,t^2+$$
$$\frac{mx\,x}{H1-r^2L\,sx^2}-\frac{my\,r\,x}{H1-r^2L\,sx\,sy}-\frac{x^2}{2\,H1-r^2L\,sx^2}+$$
$$\frac{my\,y}{H1-r^2L\,sy^2}-\frac{mx\,r\,y}{H1-r^2L\,sx\,sy}+\frac{r\,x\,y}{H1-r^2L\,sx\,sy}-\frac{y^2}{2\,H1-r^2L\,sy^2}-$$
$$t\,J-\frac{mx\,p}{H1-r^2L\,sx^2}-\frac{my}{H1-r^2L\,sy^2}+\frac{mx\,r}{H1-r^2L\,sx\,sy}+\frac{my\,p\,r}{H1-r^2L\,sx\,sy}+$$
$$\frac{p\,x}{H1-r^2L\,sx^2}-\frac{r\,x}{H1-r^2L\,sx\,sy}+\frac{y}{H1-r^2L\,sy^2}-\frac{p\,r\,y}{H1-r^2L\,sx\,sy}NN$$

$$d\,tE$$

```
step2 = Simplify@Expand@gg@x, yD@@2DDDD
```

$$\frac{E^{-\frac{\text{Hmx}-\text{my p}-\text{x}+\text{p yL}^2}{2\,\text{Hsx}^2-2\,\text{p r sx sy}+\text{p}^2\,\text{sy}^2\text{L}}} \bullet \text{!!!!!!} \, 2\,\pi}{\text{"}\;\text{\#\#\#\#sx}^2\;2\,\text{p r sx sy}+\text{p}^2\,\text{sy}^2\text{\#\#}\;\overline{\text{H}-1+\text{r}^2\text{L sx}^2\,\text{sy}^2}}$$

Then we propagate through the gap, by considering that the blur function is obtained by slicing along the line x-x0 = g y:

```
ggg@x_, x0_D =
 Simplify@Expand@ReplaceAll@gg@x, yD@@2DD, y -> Hx - x0L • gDDD
```

$$\frac{E^{-\frac{\text{Hg H}-\text{mx}+\text{my p}+\text{xL}+\text{p H}-\text{x}+\text{x0LL}^2}{2\,\text{g}^2\,\text{Hsx}^2-2\,\text{p r sx sy}+\text{p}^2\,\text{sy}^2\text{L}}} \bullet \text{!!!!!!} \, 2\,\pi}{\text{"}\;\text{\#\#\#\#sx}^2\;2\,\text{p r sx sy}+\text{p}^2\,\text{sy}^2\text{\#\#}\;\overline{\text{H}-1+\text{r}^2\text{L sx}^2\,\text{sy}^2}}$$

Some checks:

```
ggg@x, 0D
```

$$\frac{E^{-\frac{\text{H}-\text{p x}+\text{g H}-\text{mx}+\text{my p}+\text{xLL}^2}{2\,\text{g}^2\,\text{Hsx}^2-2\,\text{p r sx sy}+\text{p}^2\,\text{sy}^2\text{L}}} \bullet \text{!!!!!!} \, 2\,\pi}{\text{"}\;\text{\#\#\#\#sx}^2\;2\,\text{p r sx sy}+\text{p}^2\,\text{sy}^2\text{\#\#}\;\overline{\text{H}-1+\text{r}^2\text{L sx}^2\,\text{sy}^2}}$$

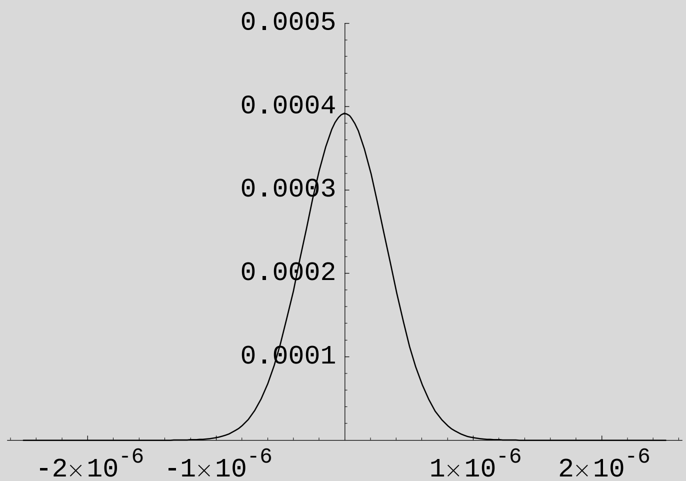
And now introduce some numerical values (in mm):

```
gplot@x_D =
 ReplaceAll@ggg@x, 0D@@2DD, 8sx -> 1, sy -> 0.00025, p -> 5000, g -> 10^-3<D
```

$$-\frac{1}{x}\left(3.52618\times 10^{-11} E^{-8.\times 10^{12} x^2}\left(-5.5509\times 10^{6} E^{3.12195\times 10^{12} x^2} x - 5.5509\times 10^{6} E^{3.12195\times 10^{12} x^2} x\, \mathrm{Erf}[1.7669\times 10^{6} x]\right)\right) + \frac{1}{x}\left(3.52618\times 10^{-11} E^{-8.\times 10^{12} x^2}\left(5.5509\times 10^{6} E^{3.12195\times 10^{12} x^2} x - 5.5509\times 10^{6} E^{3.12195\times 10^{12} x^2} x\, \mathrm{Erf}[1.7669\times 10^{6} x]\right)\right)$$

And we can plot out the result:

```
Plot@gplot@xD, 8x, -0.0000025, 0.00000250<, PlotRange -> 80, 5 10^-4<,
 PlotPoints -> 101D
```




```
- Graphics -
```

Notice that the standard deviation is about 2 10^-8; for comparison:

```
Sqrt@H1 • 5000L^2 + H0.00025L^2D 10^-2
```

3.20156×10^{-6}

```
Sqrt@H1 • 5000L^2 + H0.00025L^2D 10^-2
```

More accurately:

```
data = Table@8x, gplot@xD<, 8x, -0.0000025, 0.00000250, 0.000000101<D;
```

```
Length@dataD
```

```
50
```

```
StandardDeviation@dataD
```

81.47232×10^{-6}, 0.000116716<